

## Parameter estimation for differential problems through multi-fidelity physics-informed neural networks

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Physics-informed neural networks (PINNs) provide a deep learning framework for simultaneous reconstruction of partial differential equations (PDEs) solutions and unknown parameters from measured data. This approach has shown great potential for a wide range of problems. However, its performance may deteriorate when only partial and noisy measurements are available.

In this talk, we present a multi-fidelity and physics-based approach for parameter estimation problems, tailored also for the small data or large noise regimes. Specifically, we combine two artificial neural networks (ANNs): the first one, trained on parametrized numerical snapshots, acts as a low-fidelity solution, which is then corrected by a second physics-informed ANN. We compare two alternative strategies for training the first low-fidelity ANN, which differ on whether or not the ANN learns the parametric dependence. In the first strategy, we fix the value of the parameters to construct the dataset, while in the second one, we generate the data from samples in the parameter space (included among the ANN independent variables).

We present numerical results related to parameter estimation problems involving both benchmark test cases and systems of PDEs arising from biological problems. Our numerical tests show that the developed multi-fidelity approach improves the convergence speed of PINNs training in all cases. Moreover, the parametrized strategy increases the accuracy in estimating the unknown parameters, also in the case of a few noisy measurements.

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## REFERENCES

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