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A theoretical and numerical study on the Scaled Spherical Simplex filter with n+2 sigma points and its UKF equivalency for recursive Bayesian estimation

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The computational efficiency and performance of general Bayes filtering methodologies are largely conditional on the number of model evaluations, dictated by the number of sampling, or else sigma, points required by the filter at each time step to effectively quantify quantities of interest. In this work, a Scaled Spherical Simplex filter (S3F) is presented for recursive Bayesian estimation and nonlinear system identification, with nearly 50% reduced computational effort in all general cases, as compared to the state-of-art Unscented Kalman filter (UKF), while also achieving the same order of accuracy. In contrast to the typical $2n+1$ UKF, with n being the state space dimension, the S3F requires $n+2$ number of sigma points for the nonlinear filter transformations, thus reducing the computational demand of the filtering process. The $n+1$ Spherical Simplex filter (S2F) has the minimum possible number of sigma points needed to provide mean and nonsingular covariance estimates, but it cannot achieve the same order of accuracy and robustness as the UKF. By adding one more sigma point to this minimum set, and assigning general, well-defined weights and scaling factors, the suggested S3F can instead preserve all the important features of the UKF. A comprehensive study is thus performed in this work, investigating the filter development and performance, comparing to the standard UKF, as well as its use in system identification problems. Detailed theoretical derivations fully explain the effects of the scaling factors, sigma points location and weight selection, and prove the equivalency between UKF and S3F. Several theoretical examples are also shown, by comparing the estimated mean and covariance outputs of nonlinear functions, assuming bivariate input cases described by correlated Gaussian and lognormal random variables, as well as an arbitrarily defined joint density function. While the suggested filtering technique can be effectively employed for numerous applications, the filter performance is demonstrated here for online probabilistic inference of nonlinear dynamic structural systems. Numerical examples with increasing level of complexity are presented, to showcase the capabilities and advantages of the suggested approach, for problems associated with dual state and parameter estimation, considering nonlinear systems with hysteresis and degradations, sparsity of measurements, large observation and input noises, and time-variant parameters, among others. Emphasis is also given on the integration of the filtering framework with our recently developed damage consistent hysteretic finite element modeling approach.

REFERENCES

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