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MULTIGRID REDUCTION IN TIME FOR HIGH-ORDER ADVECTION VIA DISSIPATIVELY CORRECTED COARSE-GRID OPERATORS

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Because computer clock speeds are no longer increasing, the sequential time marching approach used in science simulation codes is becoming a bottleneck. Parallel time integration is a way of creating concurrency in a simulation that can be exploited to remove this bottleneck and provide speed ups, sometimes dramatic. The multigrid reduction in time (MGRIT) [1] approach applies existing knowledge and expertise in parallel spatial multigrid methods to the time dimension. The MGRIT method is designed to be as non-intrusive as possible and to take advantage of existing simulation codes and techniques as much as possible. This has worked well for parabolic equations, but parallel-in-time methods for advection-dominated or purely hyperbolic problems have proven to be difficult to develop [2].

In this work, we consider the application of MGRIT to linear advection PDEs. The key to efficient time integration with these methods is using a coarse-grid operator that provides a sufficiently accurate approximation to the so-called ideal coarse-grid operator. For certain classes of semi-Lagrangian discretization, we introduce a novel semi-Lagrangian-like coarse-grid operator that leads to fast multilevel time integration of variable-wave-speed linear advection. The operator is composed of a semi-Lagrangian discretization followed by a correction term that is designed such that the leading-order truncation error of the composite operator is approximately equal to that of the ideal coarse-grid operator. Parallel results show speed-ups over sequential time integration for variable-wave-speed advection problems using high-order discretizations up to order 5.

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