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Using Spectral Submanifolds for Nonlinear Control

Florian Mahlknecht[1], John Irvin Alora[1], Shobhit Jain[2], Edward Schmerling[1], Riccardo Bonalli[3], George Haller[2], Marco Pavone[1]

[1] Department of Aeronautics and Astronautics, Stanford University, Stanford, CA, 94305, USA {florianm, jjalora, schmrlng, pavone}@stanford.edu
[2] Institute of Mechanical Systems, ETH Zürich, 8092 Zürich, Switzerland {shjain, georgehaller}@ethz.ch
[3]Laboratory of Signals and Systems, University of Paris-Saclay, CNRS, CentraleSupélec, riccardo.bonalli@l2s.centralesupelec.fr

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Very high dimensional nonlinear systems arise in many engineering problems from discretization of the governing partial differential equations, e.g., through finite element methods. Despite their very successful application in, e.g., structural engineering, they have limited applicability for automatic control due to their intractable state-space size.

Previously, projection-based model order reduction techniques have been applied to obtain computationally feasible models to control such systems [2]. They build on the underlying assumption that dominant behaviors of the system dynamics are captured in the chosen linear subspace. This approach has been successfully applied in high-dimensional systems that are controlled about an equilibrium point. However, in the general nonlinear case, the projected dynamics might not preserve the full-order behavior and the subspace choice needs to be tuned in a case-by-case fashion.

Recent advances in spectral submanifold theory [1] justify model reduction under well-defined assumptions. So far, the theory allows quasi-periodic forcing with small amplitudes. In this work, we extend the setting to the presence of a control signal, actively driving the system to a desired state. We derive a computationally efficient control law that is able to close the feedback loop in real-time within an observable state-space dimension. Control performance is shown to be superior on a soft robot, where previous piecewise-affine, linearization techniques resulted in non-smooth trajectories.

This contribution paves the way for more rigorous applications of FEM models in control system design.

References

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