

Modeling and Simulation of Vascular Tumors Embedded in Evolving Capillary Networks

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We present a model for dynamically changing vascular networks whose angiogenetic growth is governed by an underlying tumor model based on [1]. The tumor model consists of 3D equations for the proliferative, hypoxic, and necrotic cells, described by Cahn-Hilliard type equations and coupled via a nutrient field to the 1D networks. On these 1D vessels, dimensionally-reduced equations for blood flow and transport are solved and coupled to 3D fields. A non-deterministic rule-based angiogenesis algorithm describes the evolution of these networks in 3D and allows the vessels to sprout, bifurcate, grow, shrink, and merge. Hence, hypoxic tumor cells can influence the vascular growth by emitting tumor angiogenesis factors, which trigger the rules for sprouting new vessels and direct the blood vessel growth towards the tumor.

The vasculature not only influences the tumor via nutrient concentrations, it also governs the transport of chemo- and immunotherapeutic drugs that kill proliferative cell populations. Thus, the vasculature is shown to have an indirect impact on patient-specific treatment protocols. Therefore, angiogenesis models such as those investigated in this work can provide vital information on the design of therapeutic treatments.

Among results described in this study are simulations involving experimentally-observed vasculature structures in mouse models [2] initially containing small spherical tumors. The evolution to a more refined vascular network that supports tumor growth is demonstrated.

REFERENCES

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