

## Three-grid immersed finite elements for complex CAD models

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In this work, we present an optimally convergent finite element method that uses the implicit description of parametric CAD models. Obtaining the implicit signed distance function for complex CAD geometries is challenging especially when they consist of free-form splines. In this case, it is expedient to approximate the exact CAD surface with sufficiently fine triangular facets, i.e., STL mesh. To compute the approximate signed distance function, we immerse the faceted surface in an adequately large cuboid discretised with a Cartesian grid of size proportional to the dimension of the surface triangles. This grid is referred to as the geometry grid. The distance function is determined by first computing the distance of the grid points from the approximate surface and then using linear interpolation within the cells. The memory footprint and efficiency of the geometry grid are improved by employing a dynamic data structure that keeps a fine resolution only within a narrow band near the boundary. In contrast, the physical field has to be finely resolved where the second derivatives of the physical field are large. Therefore, we construct an octree-based adaptive grid, referred to as the finite element grid, where the physical field is discretised using the basis functions associated with the grid cells. The partitioning of the octree data structure of the finite element grid into several processes takes into account a weighted load-balancing strategy that gives priority to the cells containing parts of the domain. To increase the integration accuracy, we introduce a fine quadrature grid obtained by refining the finite element cut-cells. We use a bottom-up octree construction algorithm to accurately capture the part of the domain within the finite element cut-cells and subsequently perform linear tessellation for distributing the quadratures. The ill-conditioned system matrix due to the small contribution from the finite element cut-cells is improved by extrapolating the associated degrees of freedom from the nodal coefficients of the cells inside the domain. The parallel system is solved using the iterative preconditioned conjugate gradient (PCG) with the BDDC preconditioner. The robustness and accuracy of the developed approach are demonstrated with a number of benchmark examples and industrial geometries.