

# Refinement of polygonal grids using Convolutional Neural Networks with applications to polygonal Discontinuous Galerkin and Virtual Element methods

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We propose new strategies to handle polygonal grids refinement based on Convolutional Neural Networks (CNNs). We show that CNNs can be successfully employed to identify correctly the “shape” of a polygonal element so as to design suitable refinement criteria to be possibly employed within adaptive refinement strategies. We propose two refinement strategies that exploit the use of CNNs to classify elements’ shape, at a low computational cost. We test the proposed idea considering two families of finite element methods that support arbitrarily shaped polygonal elements, namely Polygonal Discontinuous Galerkin (PolyDG) methods and Virtual Element Methods (VEMs). We demonstrate that the proposed algorithms can greatly improve the performance of the discretization schemes both in terms of accuracy and quality of the underlying grids. Moreover, since the training phase is performed off-line and is independent of the differential model the overall computational costs are kept low. Some results will also be shown in the three-dimensional case.

## REFERENCES

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