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Application of a modified multigrid waveform relaxation method as a time-simultaneous approach to convection-diffusion equations

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We present a time simultaneous multigrid method [1] that is closely related to multigrid waveform relaxation (cf. [2, 3]). The presented scheme is motivated by reordering an all-at-once system consisting of multiple time steps to a time-major ordering. If we interpret the values of one spatial degree of freedom at all discrete time steps as a single vector-valued unknown, this results in a system that formally has the same structure as the system in the stationary case. Then, a standard geometric multigrid method is applied to this system in a block-wise fashion. Using this approach it is reasonable to apply strong smoothers like a GMRES or a BiCGSTAB method with a block-Jacobi preconditioner which has several advantages in practical applications.

While this method is not inherently parallel in time, significant speedup compared to a sequential time stepping approach can be observed due to an increased parallelization potential in space. The potential of the proposed solution technique is illustrated in numerical studies for the convection-diffusion equation, particularly arising as subproblems in the nonstationary incompressible Navier-Stokes equations. Furthermore, problems in configurations with small diffusion coefficients and possible mitigations will be discussed.

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