

SURROGATES IN PDE-CONSTRAINED ONE-SHOT OPTIMIZATION UNDER UNCERTAINTY

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Many problems in science and engineering can be formulated as optimization problems constrained by partial differential equations (PDEs) with random coefficients. Thereby the input random field is typically parametrized by a countably infinite number of parameters, and hence a very large number of PDE-solves is needed to obtain an accurate and robust solution of the underlying optimization problem.

Our approach replaces the computationally intensive solution of the parametric PDE by a surrogate, e.g., a neural network, which is learned simultaneously in a one-shot sense when solving the optimal control problem. We reformulate the problem as a penalized empirical risk minimization problem for which we provide a consistency analysis in terms of increasing sample size and increasing penalty parameter. To solve the resulting problem, we suggest a stochastic gradient method with adaptive control of the penalty parameter and prove convergence under suitable assumptions on the surrogate model. Numerical experiments illustrate the results for linear and nonlinear surrogate models.

REFERENCES

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