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VARIATIONAL PHYSICS INFORMED NEURAL NETWORKS: AN A PRIORI ERROR ESTIMATE

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Inspired by the success of deep learning in numerous engineering contexts, a novel numerical technique to solve partial differential equations has been proposed in [1]: the Physics Informed Neural Network (PINN).

In this talk, we focus on a specific class of PINNs called Variational Physics Informed Neural Network (VPINNs) [2]. Such networks are trained using exclusively the variational formulation of the equation, while standard PINNs exploit its strong formulation or known numerical solutions.

In particular, we prove an a priori error estimate for numerical solutions of second order elliptic problems [3]. Here we assume the VPINN to be a fixed-size feed forward fully connected neural network and we let vary the mesh used during the training phase. Such an estimate is useful to describe the relationship between the accuracy of the approximated solution, in H^1 norm, and the resources required to train the network.

To formally prove the error estimate, we introduce a piecewise interpolant of the VPINN to satisfy the inf-sup condition. Nevertheless, numerical experiments show that the predicted converge rate is very close to the empirical one even without such an interpolation.

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