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Convergence of a finite-volume method with strong-weak imposition of boundary conditions

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We consider a node-centered finite-volume discretisation of the heat equation, augmented with Dirichlet, Neumann and Robin boundary conditions. In particular, we study the finite-volume approximation of the Laplacian operator provided in [1]. The boundary conditions are imposed using a strong-weak combination (see [2]), where the Dirichlet condition is imposed strongly by injection while the Neumann and Robin conditions are imposed wakly using Simultaneous-Approximation-Terms (SATs). By using the Summation by Parts (SBP) properties of the Laplacian operator, stability of the scheme is proved by the energy method. We show that the numerical solution converges weakly to a unique weak solution. Subsequently, we apply Aubin-Lion's lemma to prove strong convergence. The motivation for this work is the difficulty of finding consistent finite-volume approximations of the Laplace operator. Numerical simulations are provided to corroborate the theoretical results.

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