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Decoupling time discretization methods for coupled elliptic-parabolic systems

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The classical Biot poroelasticity model is a coupled system of elliptic and parabolic partial differential equations. State-of-the-art numerical time integration schemes for this system include the robust monolithic implicit Euler discretization (e.g. [1]). For the use of specialized preconditioners for reducing the computational complexity, the fixed-stress method and the undrained splitting method (e.g. [2]) enable iterative decoupled solves. The performance, however, is strongly tied to tuning additional stabilization parameters. The accelerated non-iterative decoupled solve via the semi-explicit Euler discretization [3] is possible for a restrictive class of problems, namely if the coupling is sufficiently weak.

We first compare the run-time complexities of these four first-order methods with respect to preconditioners for the monolithic solve and the decoupled solves. Next, we explore convergence of higher order semi-explicit decoupling schemes based on implicit Runge-Kutta methods by employing the novel idea of an implicit discretization of auxiliary delay differential equations [3].

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