The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

Scalable statistical finite elements via partial differential equation representation of Matérn fields

Kim Jie Koh[1], Eky Febrianto[1,2] and Fehmi Cirak[1*]

[1] University of Cambridge,
7a JJ Thomson Avenue, Cambridge CB3 0FA, United Kingdom.
kjk46@cam.ac.uk, efebrianto@turing.ac.uk, f.cirak@eng.cam.ac.uk
[2] The Alan Turing Institute
96 Euston Rd, London NW1 2DB, United Kingdom.
efebrianto@turing.ac.uk

Keywords: *Statistical finite element method, Matérn kernel, fractional stochastic partial differential equations, sparse precision matrices*

The increased availability of observation data from engineering systems in operation poses the question of how to incorporate measurement data into finite element models. The recently proposed statistical finite element construction (statFEM) provides a principled means to synthesise measurement data and finite element models [1]. The uncertainties present in the data, the mathematical model and its finite element discretisation are taken into account using a Bayesian statistical framework. The posterior densities of the finite element solution, model misspecification error and the noise are inferred from the data by updating their respective priors. The corresponding likelihood function depends on the data and the finite element model. In the present work, we assume that all the random fields are Matérn fields and parameterise them using their fractional stochastic partial differential equation (SPDE) representation leading to sparse precision matrices. Consequently, all the precision matrices are sparse, and the precision matrix of the posterior can be obtained using only sparse matrix operations. In addition to improving scalability, the SPDE representation provides a reasonable description of covariance structures on non-Euclidean domains, accounts for smoothness variability in the data and naturally extends to non-stationary random fields. We demonstrate the scalability and efficacy of the proposed approach with several examples from structural mechanics.

REFERENCES

[1] Girolami, M., Febrianto, E., Yin, G., & Cirak, F. (2021). The statistical finite element method (statFEM) for coherent synthesis of observation data and model predictions. *Computer Methods in Applied Mechanics and Engineering*, 375, 113533.