

An optimally convergent smooth blended B-spline construction for unstructured quadrilateral and hexahedral meshes

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Easy to construct and optimally convergent generalisations of B-splines to unstructured meshes are essential for the application of isogeometric analysis to domains with non-trivial topologies. Nonetheless, especially for hexahedral meshes, the construction of smooth and optimally convergent isogeometric analysis basis functions is still an open question. We introduce a simple partition of unity construction that yields smooth blended B-splines, referred to as SB-splines, on unstructured quadrilateral and hexahedral meshes [1]. To this end, we first define the mixed smoothness B-splines that are C^0 continuous in the unstructured regions of the mesh but have higher smoothness everywhere else. Subsequently, the SB-splines are obtained by smoothly blending the mixed smoothness B-splines with Bernstein bases of equal degree. One of the key novelties of our approach is that the required smooth weight functions are assembled from the available smooth B-splines on the unstructured mesh. The SB-splines are globally smooth, non-negative, have no breakpoints within the elements and reduce to conventional B-splines away from the unstructured regions of the mesh. Although we consider only quadratic mixed smoothness B-splines, the construction carries over to arbitrary degrees. We demonstrate the excellent performance and optimal convergence of SB-splines studying Poisson and biharmonic problems.

REFERENCES

[1] Koh, K. J., Toshniwal, D., & Cirak, F. (2021). An optimally convergent smooth blended B-spline construction for unstructured quadrilateral and hexahedral meshes. arXiv preprint arXiv:2111.04401.