

A framework for identifying better cubature rules for plane and solid elements

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As simulation requirements on the performance of computational methodologies increase, higher-order interpolations, and therefore higher-order integration schemes, can be advantageous. In the past few decades, a lot of new Gaussian type cubature rules have been found [1], but their performance inside practical Finite Element simulation is hardly explored. It is therefore meaningful to identify the new rules that have better performance than the classic integration schemes, especially the rules that behave well in the simulations which have problems like locking and spurious zero energy mode. However, in the past, only few works [2, 3] have concentrated on the tests of numerical integration schemes and none of them have developed a systematic way to individually examine numerical integration schemes.

Our purpose is to identify new cubature rules with better performance through a novel framework which, for the first time, systematically examines the performance of cubature rules in plane and solid elements. Before tests, target rules are carefully selected and classified according to its number and distribution of integration points. The framework has four steps: eigenvalue analysis, patch test, beam/plate/shell tests and stability/accuracy analysis. The third step involves only representative tests that reflects the effect of integration schemes and thus requires fewer tests than a general framework designed for elements. Besides, Displacement Stability Plot and Pathological Tests Score are employed as qualitative and quantitative comparison methods. Based on this new framework, comparing the new rules with the classic full/reduced/selective reduced integration rules, cubature rules with better performance could be identified.

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