The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

# Implicitly extrapolated geometric multigrid for the gyrokinetic Poisson equation

**Carola Kruse[1,*], Martin Kühn[2], Philippe Leleux[1], Ulrich Rüde [1,3], Christina Schwarz[3]**

[1] Cerfacs, 42 Av Gaspard Coriolis, 31057 Toulouse, carola.kruse@cerfacs.fr, philippe.leleux@cerfacs.fr
[2] DLR, Linder Hoehe, 51147 Cologne, martin.kuehn@dlr.de
[3] FAU, Cauerstr. 11, 91058 Erlangen, ulrich.ruede@fau.de, christina.schwarz@fau.de

**Keywords**: *multigrid, implicit extrapolation, fusion plasma simulation, matrix-free*

We are interested in the solution of the gyrokinetic Poisson equation appearing as part of a 5D coupled problem in the context of Tokamak plasma simulations. The gyrokinetic Poisson equation is solved on a large number of poloidal cross sections of the Tokamak geometry; see, e.g., [1]. In the current Gysela simulation code, this task represents a bottleneck with respect to the computation time and thus requires fast and scalable solvers.

In our work, we make use of geometric multigrid methods, as these can achieve optimal complexity and are among the most efficient solvers for elliptic partial differential equations. As a starting point, the cross sections can be considered as circular domains, but deformed geometries were found to be advantageous and more realistic for the Tokamak application. However, multigrid methods for geometries described by curvilinear coordinates are less common. In this talk, we present a tailored geometric multigrid algorithm using optimized line smoothers to enable parallel scalability [2]. We use a finite difference discretization of the energy potential of the Poisson equation, so that a matrix-free implementation with a low memory footprint is possible. Furthermore, we propose an implicit extrapolation technique that increases the order of convergence from linear to at least quadratic order. Finally, we will present scalability results for our recent OpenMP and Cuda implementations of the solver.

This work was funded from the European Union's Horizon 2020 research and innovation programme under grant agreement no. 824158.

## REFERENCES

[1] N. Bouzat, C. Bressan, V. Grandgirard, G. Latu, M. Mehrenberger, *Targeting Realistic Geometry in Tokamak Code Gysela.* ESAIM: ProcS, 63, pp. 179-207, 2018.

[2] M. Kühn, C. Kruse, U. Rüde, *Energy-minimizing, symmetric finite differences for anisotropic meshes and energy functional extrapolation*, SIAM J. Sci. Comput., 43(4), A2448–A2473, 2021