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A collocation method based on single-layer feedforward neural network for the resolution of Elliptic PDEs.

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In this talk, we present the construction of a collocation method with a Feedforward Neural Network (FNN) with a single hidden layer and sigmoidal transfer functions randomly generated, the so-called Extreme Learning Machines (ELM): we use ELM random projection networks as discrete space where to look for the solution of PDEs. Free parameters (N external weights) are fixed by imposing exactness on M (eventually located randomly) points via collocation. In order to obtain accurate solutions, we underdeterminate the collocation equations ($N > M$). For linear PDEs, the weights are computed by a one-step least-square solution of the linear system. The least-square solution is capable of automatically selecting the important features, i.e. the functions in the space that are more influent for the solution. This leads to a one-shot automatic method and there is no need for adaptive procedures or tuning of the parameters as done when learning in other methods based on FNN.
We present results on elliptic problems both in the linear case and for the resolution and construction of bifurcation diagrams of nonlinear problems. For linear elliptic PDEs, in [1] we introduce the method and prove that ELMs can adapt "automatically" via Least Square and gain good accuracy also in steep gradient benchmark problems. In the case of nonlinear elliptic PDEs, in [2] we extend the method in order to deal with the nonlinearity and prove accuracy when computing tuning points of bifurcation diagrams. Burgers and Bratu's equations are taken as benchmark nonlinear problems.
The results are obtained in collaboration with Gianluca Fabiani and Costantinos Siettos.

REFERENCES

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