# Conservative stochastic reduced order models for real-time fluid flow data assimilation

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Fluid dynamics intrusive reduced order models (ROM) – such as Proper Orthogonal Decomposition (POD)-Galerkin ROM – often suffer from important truncation errors. Furthermore, for long-time integrations, these errors grow possibly without bound. Closures alleviate truncation errors but without preventing divergences in long-time extrapolations. Additionally, for data assimilation purposes (e.g., synchronized real-time digital twin), ROM solution errors need to be accurately quantified.

In order to address these issues, we rely on a conservative stochastic closure, referred to as model under location uncertainty [1]. Rigorously derived through stochastic calculus tools, the proposed framework encompasses several meaningful mechanisms for turbulence modelling. In particular, a turbulent dissipation always generates a covariance inflation – through correlated additive and multiplicative noises – and the energy is conserved.

This framework is of particular interest for low-order representations. The deterministic ROM coefficients are obtained by a Galerkin projection whereas the correlations of the noises are estimated from the residual velocity, the model structure, and the evolution of the resolved modes. Our stochastic model reproduces the triades between temporal modes either in a deterministic or in a random way – depending on the number of resolved modes [1]. These energy transfers stabilize unstable modes and maintain the variability of stable modes.

This reduced uncertainty quantification and data assimilation procedure has been applied to a two- and a three-dimensional wake flows respectively. The ROM shows very good uncertainty quantification and prediction skills [1]. Without initial condition information, our on-line data assimilation technique assimilates measurements on-the-fly [2]. As a benchmark, the same procedure was done with a reduced order model with optimally fitted eddy viscosity and additive noise, and our method goes far beyond its flow estimation capabilities. Our methodology is now implemented on the OpenFOAM-based ROM library ITHACA-FV [3].

## REFERENCES

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