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Bound-preserving and entropy-stable algebraic flux correction schemes for the shallow water equations with topography

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Numerical methods for solving hyperbolic problems need to be carefully designed to obtain physically correct approximations. Examples of such considerations for the shallow water equations, are well-balancedness, nonnegativity of water heights, and, entropy stability. Recent years saw an increased interest in algebraic flux corrections schemes based on (dis-)continuous finite element methods. Such bound-preserving [1] and entropy-stable [3] approaches for continuous finite elements have been successfully applied to the shallow water equations with flat bottom topography [2].

In this work, we extend the above techniques to the shallow water system with topography source term. Our method preserves the lake at rest up to machine precision. Nonnegativity of water heights can be proven under a standard CFL condition, and a semi-discrete entropy inequality is shown. Our scheme can be interpreted as a generalization of certain finite volume schemes to the finite element setting. Numerical examples for well-known benchmarks are presented to evaluate the performance of the scheme. In particular, I will discuss some examples where wetting-and-drying algorithms need to be employed.

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