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Asymptotically based simulation of the Stokes flow in a layer through periodic flexural plates made of beams

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Stokes fluid is flowing through a spacer fabric, a three dimensional textile structure which can be modeled by a porous layer between two parallel hyperplanes with periodically distributed parallel beam lattices, which are orthogonal to the hyperplanes. The flow direction is parallel to the hyperplanes and orthogonal to the lattices. The top and the bottom of the spacer fabric is insulated for the in- and outflow. The thickness of the spacer fabric is assumed to be one, while the thickness of the lattices or porous layers is a small parameter ε. The fluid viscosity is assumed to be of order $\varepsilon^3 E$, where E is the Young's modulus of the beams.

Fluid-solid interaction is considered in the structure; the deflection of the beam lattices due to pressure jumps in the fluid is of interest for the practical application. A dimension reduction as the lattice thickness $\varepsilon \to 0$ is presented in [1]. The lattice-layers are replaced by their homogenized mean surfaces leading to Stokes flow with a non-standard interface condition: the pressure jump at the mean surfaces is proportional to the biharmonic operator in the surfaces applied to the velocity trace. In the limit, the normal component of the macroscopic velocity field is an H^2-function of the lattice mean-plane variable and the limit problem is non-local in time. This corresponds to the non-stationarity of the initial problem.

The dimension reduction approach enables a decoupling of the numerical solving process into two main steps, drastically reducing computational effort compared to the initial fluid-solid interaction problem with the fully resolved lattice structures.

In the first step, effective outer-plane bending and in-plane tension coefficients of the beam lattices are computed. The computation follows a further dimension reduction (see [2],[3],[4]), reducing the elasticity problem to beam equations on one-dimensional networks and further to a linear algebraic system with 6 unknown degrees of freedom in the nodes of the network.

Afterwards, the non-local limit problem is solved by application of a spatial finite element approach. The 3D-domain is efficiently discretized by a hexaedral mesh using linearly extended Bogner-Fox-Schmit elements for the normal component of the fluid velocity field. The approach results in a linear integro-differential equation which is reformulated to an equivalent stiff linearly implicit ODE with a total of 12 degrees of freedom per mesh node, representing fluid velocity and interface displacements in three dimensions.

Convergence estimates from the corresponding analysis will be used to estimate the numerical accuracy of the reduced dimension algorithm. Finally, local stresses in the beams and the fluid pressure will be reconstructed as in [2] with the help of interpolated and extended piece-wise polynomial function sequences which are strongly convergent to the solution.

The derived method finds application in the simulation of filter properties for spacer fabrics and their optimization with regards to structural design parameters.

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