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Positivity-Preserving Entropy-Based Adaptive Filtering for Discontinuous Spectral Element Methods

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The use of discontinuous spectral element methods (DSEM) has grown in prevalence over the years due to their ability to achieve high-order accuracy while retaining geometric flexibility. As a result, DSEM offer many advantages for simulations of complex problems. However, their robustness is severely reduced for problems that exhibit discontinuities as the presence of spurious oscillations can result in nonphysical solutions or the failure of the scheme altogether. Consequently, this lack of reliability is one of the limitations preventing the widespread adoption of these methods in the industry.

To extend the use of DSEM to a wider variety of problems, various stabilization techniques have been proposed to increase the robustness of these schemes in the vicinity of discontinuities. A common goal of these shock capturing methods is to suppress numerical instabilities in the vicinity of a shock without degrading the accuracy of the underlying numerical scheme in regions where the solution is smooth. However, a drawback of many of these approaches in general is that they (1) do not necessarily guarantee that physical constraints on the solution are satisfied, which may lead to the failure of the scheme, (2) present free parameters in the implementation which can require problem- and mesh-dependent tuning, a cost that cannot be afforded for large scale-resolving simulations, and/or (3) are not easily and efficiently implemented in the context of explicit DSEM on modern computing architectures. These issues motivate the development of shock capturing approaches for DSEM that can guarantee certain physical constraints are satisfied without requiring tunable parameters.

In this talk, we will present adaptive filtering approach for shock capturing in DSEM to address these issues. By formulating physical constraints such as positivity and a local minimum entropy principle as constraints on the discrete solution, the filter strength is computed via a simple scalar optimization problem requiring only element-local information. Under some basic assumptions on the properties of the numerical scheme, the filtered solution is guaranteed to satisfy these constraints, resulting in an efficient but robust method for resolving discontinuous features without the use of problem-dependent tunable parameters. Furthermore, the proposed filtering approach recovers the standard DSEM approach for smooth solutions, retaining the efficiency, accuracy, and geometric flexibility of the underlying scheme. The efficacy of the approach will be shown in numerical experiments on hyperbolic and mixed hyperbolic/parabolic conservation laws such as the Euler and Navier-Stokes equations for problems including extreme shocks, shock-vortex interactions, and complex compressible turbulent flows.