

The Inverse Finite Cell Method for Structural Identification

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Key Words: *Finite Cell Method, Wave Equation, Inverse Problems, Full Waveform Inversion, Non-destructive testing*

Immersed Boundary Methods like the Finite Cell Method [1] or the Spectral Cell Method [2] use an indicator function α to separate the physical domain of computation from an embedding, fictitious domain Ω_{fict} . The Ansatz functions are defined on a domain-independent background mesh being defined on the union of both domains. Elements cut by the boundary are thus characterized by a jump in the indicator function, requiring special techniques for the integration of discontinuous functions.

In classical boundary value problems, the indicator function α is defined *a priori* by the geometry. A similar principle of separating geometry description and element shapes can be used for an inverse problem, where α itself is an unknown and acoustic or elastic wave propagation is considered. By discretizing α on the finite cell grid, we obtain a finite number of unknowns that we determine by minimizing an objective function that compares measured transient wave signals to the intact model-based response. As this naturally leads to a large number of optimization parameters, we use an adjoint approach for computing gradients. This idea essentially transfers the material descriptions known from topology optimization to the applications of non-destructive testing using a full waveform inversion (FWI) formulation.

The combination of FCM with FWI offers several advantages over FEM- or FD-based methods. First, we can incorporate *priori* knowledge of even very complex geometries via the immersed approach, and second, an optimization w.r.t. the indicator function α allows for high contrast of material parameters, i.e. the detection of voids or cracks. We describe the principles of this approach and show its suitability for problems of non-destructive testing on several examples.

REFERENCES

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