

# Stress minimization for lattice structures

A. Ferrer<sup>\*3,4</sup>, P. Geoffroy-Donders<sup>2</sup>, and G. Allaire<sup>1</sup>

<sup>1</sup>CMAP, École Polytechnique, Institut Polytechnique de Paris, 91128 Palaiseau, France.

<sup>2</sup>Arts et Métiers Paristech, 151, Boulevard de l'hôpital, 75013 Paris, France

<sup>3</sup>CIMNE - International Center for Numerical Methods in Engineering, Campus Nord UPC, Edifici C-1, c/Jordi Girona 1-3, 08034 Barcelona, Spain

January 2022

## Abstract

In the recent years, additive manufacturing has allowed to build complex geometries with a small minimum length scale (lattice structures). This manufacturing revolution has directly impacted on the field of topology optimization, enabling the classical homogenization techniques to resurrect. This is the case of the recent work [1] where optimal lattice structures have been obtained when minimizing compliance. In this work, we follow the techniques developed in [1] but we focus on controlling the stress norm of the structure, one of the key aspects in structural design. In this case, the main idea is to consider in the homogenization theory the stresses appeared in the micro-structure. For that propose, the macroscopic stresses must be corrected via the amplifier tensors. Thus, we extend the results obtained in [2] (rank-q laminates) for the case of lattice structures. As in work [1], the micro-structure is described by two geometrical parameters and the orientation of the cell. However, in this case, we propose a smooth geometry of the micro-structure to reduce stress concentration. The optimization process is divided in three steps. Step 1. For a wide number of parameters, we compute and save the homogenized material properties of the micro-structure (amplifiers, homogenized constitutive tensor and fraction volume). We call this pre-computed data, once it is linearly interpolated, computational Vademeum. Step 2. We consider the stress norm and the volume as cost function and constraint of the optimization problem and we use a standard projected gradient for updating the micro-structural parameters. For the orientation of the cell, we propose to align it with the principal stress direction following the results obtained in [1] for the case of the compliance. This choice comes from an accurate analysis of the amplifiers components. Step 3. Finally, we proceed with the de-homogenization process [1]. Basically, it consists in the description of the geometry via a level-set function which depends on the coordinates and the microscopic parameters. The first depends on the microscopic coordinate  $y = x/\varepsilon$  while the second on the macroscopic coordinate  $x$ . Subtle geometrical techniques are considered for avoiding singularities in the orientation field. The work ends showing several numerical results. First, the values of the amplifiers for the proposed smooth microscopic geometry will be compared with a non-smooth one. Finally, the optimization process for both cases and the optimal de-homogenized structures will be shown [3].

## References

- [1] Grégoire Allaire, Perle Geoffroy-Donders, and Olivier Pantz. Topology optimization of modulated and oriented periodic microstructures by the homogenization method. *Computers & Mathematics with Applications*, 78(7):2197–2229, 2019.
- [2] Grégoire Allaire, François Jouve, and Hervé Maillot. Topology optimization for minimum stress design with the homogenization method. *Structural and Multidisciplinary Optimization*, 28(2-3):87–98, 2004.
- [3] Alex Ferrer, Perle Geoffroy-Donders, and Grégoire Allaire. Stress minimization for lattice structures. Part I: Micro-structure design. *Philosophical Transactions of the Royal Society A (2021)*, 2020.

---

<sup>\*</sup>aferrer@cimne.upc.edu