The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5 – 9 June 2022, Oslo, Norway

Numerical Solution of 3D Diffusion Equations Via a Non-Linear Flux Limited Splitting Using the Multipoint Flux Approximation Method with a Diamond Stencil Satisfying the Discrete Maximum Principle

Artur Castiel Reis de Souza[1a], Darlan Karlo Elisiário de Carvalho[1b], Túlio de Moura Cavalcante[1c], Filipe Antônio Cumaru Silva Alves[1d], Michael G. Edwards[2] and Paulo Roberto Maciel Lyra[1e]

[1] Universidade Federal de Pernambuco, Av. da Arquitetura, S/N, CEP 50740-550 Recife, Brazil,
[a]artur.castiel@ufpe.br, [b]darlan.ecarvalho@ufpe.br, [c]tulio.cavalcante@ufpe.br,
[d]filipe.cumaru@ufpe.br, [e]paulo.lyra@ufpe.br
[2]Swansea University, Bay Campus, Swansea, SA1 8EN, UK, m.g.edwards@swansea.ac.uk

Key Words: *MultiPoint Flux Approximation, Least Square Conservative Weight Nodal Interpolation, Non-Linear Flux Limited Splitting, DMP, Unstructured Grids, Anisotropic 3-D Diffusion Problems.*

In the present paper, we solve the steady state diffusion equation in 3D domains by means of a cell-centred finite volume (FV) method employing a Multipoint Flux Approximation with a Diamond (MPFA-D) stencil using, for the first time in literature, a least square conservative weight nodal interpolation in 3D with a Non-Linear Flux Limited Splitting (NL-FLS) which is capable of satisfying the Discrete Maximum Principle (DMP). Our formulation is based on the M-Matrix Flux splitting that separates the MPFA flux in terms of the linear Two Point Flux Approximation (TPFA) flux and the Cross-Diffusion Terms (CDT), but with the addition of a strategy for detecting local loss of DMP and for computing a relaxation parameter for the CDT providing just an acceptable amount of cross diffusion to eliminate the DMP violation. Our scheme eliminates non-physical spurious oscillations intrinsic to linear MPFA schemes by bringing the flux approximation closer to the TPFA in these regions. Moreover, to ensure that our formulation is truly cell-centred, we use a linearity-preserving (LP) robust weighted Global Least Square (GLS) interpolation. Our resulting framework is locally conservative and capable of dealing with arbitrary anisotropic diffusion tensors and unstructured meshes, producing solutions that satisfy the DMP, achieving second order accuracy for the scalar variable in smooth fields and first order accuracy for fluxes. To assess the accuracy and robustness of our formulation, we solve some challenging benchmark problems and the results are compared to other linear formulations.

REFERENCES

[1] C. Dong, and T. Kang, "A least squares based diamond scheme for anisotropic diffusion problems on polygonal meshes", *Int. J. Numerical Methods in Fluids*. 2021, 1–23, DOI: 10.1002/fld.5031.
[2] R. J. M. Lira Filho, , S. R. Santos, T. M. Cavalcante, F. R. L. Contreras, P. R. M. Lyra, D. K. E. Carvalho. A Linearity-Preserving Finite Volumes Scheme with a Diamond Stencil for the Simulation of Anisotropic and Highly Heterogeneous Diffusion Problems using

Tetrahedral Meshes, *Computers & Structures*, 2021,Vol. 250, pp. 106510. DOI: 10.1016/j.compstruc.2021.106510.
[3] M. Pal and M.G. Edwards. "Non-linear flux-splitting schemes with imposed discrete maximum principle for elliptic equations with highly anisotropic coefficients". *International Journal for Numerical Methods in Fluids*, Vol. 66, 2011. DOI: 10.1002/fld.2258.
[4] A.C.R. Souza, T.M. Cavalcante, M.G. Edwards, D.K.E. Carvalho, P. R. M. Lyra, "Numerical Simulation of the Diffusion Equation Via a Non-Linear Flux Splitting Technique with the Multipoint Flux Approximation Method with a Diamond Stencil Satisfying the Discrete Maximum Principle Using 2-D Unstructured Meshes", *proceedings of COB*EM2021, Florianópolis/SC, November, 2021.