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Generalization of hydrodynamic sea ice dynamics using kinetic theory and the Boltzmann equation

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We evolve a distribution representing the probability sea ice is at position x with velocity v using the Boltzmann equation. Sea ice dynamics are driven by oceanic/atmospheric currents and small-scale collisions between ice floes—chucks of ice. Hydrodynamic sea ice models treat sea ice as a fluid-like continuum, modeling averaged quantities, such as ice velocity, over a representative volume element. We show typical conserved quantities—mass, momentum, and energy—correspond to moments of the probability density. We also show that the hydrodynamic models correspond to the limit of the non-dimensional mean free path parameter being zero. Intuitively, the mean free path is the ratio of the largest distance an ice floe travels before colliding with another floe to the domain length. The hydrodynamic limit, therefore, corresponds to arbitrarily small ice floes that are in constant contact with their neighbors. However, there is no characteristic floe size. Data suggest the floe size distribution follows a power law and, therefore, improving large-scale models requires a multi-scale approach. Solving the Boltzmann equation with non-zero mean free path generalizes the hydrodynamic limit, allowing us to account for larger ice floes and less densely packed regions. Since the Boltzmann equation is notoriously difficult to solve numerically, we devised a superparameterized multi-scale approach that combines large-scale conservation equations with small-scale kinetic models. In particular, we use a discontinuous Galerkin method to solve the mass and momentum conservation equations and particle methods associated with key points in the domain that we use to estimate higher-order moments. Importantly, our method *independently* updates the particle methods at each timestep, allowing us to leverage high performance computing and parallel resources.