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VARIATIONAL APPROACHES TO VARIOUS NUMERICAL SCHEMES FOR FLUID AND MULTI-FLUID FLOWS WITH GEOMETRY–ENERGY–ENTROPY COMPATIBILITY

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Since WW2, computer fluid dynamics has seen a staggering expansion of methods and applications fueled by the development of computer power. Now, of the main numerical approaches that have been explored over these years, only a few have become mainstream and make the vast majority of theoretical investigations in academia and practical usage in applications. These mostly hinge on concepts of finite volume discretization, monotonicity preservation, flux upwinding, and the analysis of the associated numerical dissipation processes—common tools here are the Riemann problem at cell interfaces and the Godunov scheme, more or less adapted from their original versions.

However, among other issues, isentropic evolution is of high practical interest in compressible flows but is generally poorly captured by overly dissipative Riemann based solvers. Correcting schemes for excessive dissipation turns out to be somewhat convoluted and sometimes fragile. The issue can become very complex in multi-fluid systems where errors in pressure coupling terms translate in possible violations of the second principle of thermodynamics: for N fluids, there are N entropy conditions and at least $2N$ energy reservoirs (kinetic and internal) coupled through $N(2N-1)$ pressure terms. For $N=10$, a modest number of fluids in various important systems, this makes 190 coupling terms to be consistently discretized with minimal but positive dissipations.

The present contribution aims at providing some perspective on CFD numerical schemes recently designed in order to better capture isentropic flows. The basic principle is that isentropic flow is *geometric*, i.e. potential (or internal) energy only depends on fluid density which in turn is defined by fluid element trajectories. A numerical scheme can thus be obtained by a variational, least action principle. Corrections must be further added to enforce other properties such as energy conservation and positive dissipation. This Geometry, Energy, and Entropy Compatible approach (GEEC) [1, 2] is illustrated here on a recently developed multi-fluid compressible scheme in an Arbitrary Lagrangian–Eulerian setting (ALE) [3].

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