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General mesh discontinuous Galerkin methods and adaptivity

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In recent years, we have extended the applicability of the interior-penalty discontinuous Galerkin (dG) method to meshes comprising extremely general elements [1]. From this, we have started to investigate a further extension to curved elements and the a posteriori error analysis of the method in [2]. Therein, the dG method is used to approximate an elliptic interface problem involving, possibly, curved interfaces, with flux-balancing interface conditions. *General curved* element shapes allows exact resolution of the interface, but the approach also applies to the case of curved domains with non-essential boundary conditions. Exact representation of the problem domain allowed us to analyse an posteriori error estimator, proving upper and lower bounds of the error in the respective dG-energy norm. Subsequently, we have further generalised dG to include extremely general *curved* element shapes throughout the domain [3], hence also including the treatment of essential boundary conditions. The feasibility of the method relies on the definition of a suitable choice of the discontinuity penalization, which turns out to be explicitly dependent on the particular element shape, but essentially independent on small shape variations. Based on these works, we will present a new *a posteriori* error analysis which applies to meshes consisting of very general polygonal/polyhedral elements, including elements with arbitrary number of faces, as long as certain mild shape regularity assumptions are satisfied. The case of simplicial and box-type elements is included as a special case, thus generalising known reults for dG methods to admit arbitrary number of irregular hanging nodes per element. The proof hinges on a new recovery strategy in conjunction with a generalized Helmholtz decomposition formula. The resulting *a posteriori* error bound involves jumps on the tangential derivatives along elemental faces along the standard residual error estimator terms.

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