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KINETIC-ENERGY INSTABILITY OF FLOWS WITH SLIP BOUNDARY CONDITIONS

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Linear stability theory is commonly known to be flawed as it predicts stability of Couette flow for all Reynold's numbers, contrary to experimental and computational evidence. In this talk, we utilize the notion of kinetic energy instability due to Reynolds and Orr [1] to determine the most unstable modes for flow around a cylinder. This method provides an exact, nonlinear instability criterion that rigorously predicts exponential growth of perturbations.

On the cylinder boundary we impose Navier slip boundary conditions, which express a balance between the amount of slip and the shear stress [2]. The Navier slip condition is attractive to work with as it is physically more accurate and computationally easier to impose. We show that for a particular value of the friction coefficient, this boundary condition gives a straightforward resolution of D'Alembert's paradox. For sufficiently large friction coefficients, the Navier slip boundary condition converges to the Stokes no-slip boundary condition.

We compute numerically the eigenproblem determining the most unstable modes for flow past a cylinder. These correspond to modes previously observed in fluid dynamic simulations, confirming the previous observations. We also identify a scaling rule regarding the critical viscosities at which instabilities get activated. Thus we can predict with some confidence the form of the most unstable modes for even larger Reynold's numbers than our current computations support. Finally, the most unstable modes are found to be rotations supported aft of the cylinder. This is in concert with what was observed in [3]. Thus our findings provide support for the observations in [3] based on rigorous mathematical theory and computations.

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