

## Exact Nonlinear Model Reduction via Direct Computation of Spectral Submanifolds in Finite Element Problems

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The prediction of a steady-state response to an externally applied dynamic load is of special significance in engineering applications. Mechanical structures are usually characterized by light damping which results in exceedingly long integration times before a steady state is reached. Despite the broad availability of dedicated software packages, the computation and continuation of the steady-state in response to periodic forcing remains a serious computational challenge for full-scale nonlinear finite element models.

The recent theory of Spectral Submanifolds [1] has laid the foundation for a rigorous model reduction of such nonlinear systems, leading to reliable steady-state response predictions within feasible computation times. Further developments [2] have enabled the computation of SSMs and their reduced dynamics by solving the associated invariance equations directly in physical coordinates using only the eigenvectors associated with the master modal subspace. The software implementation of the method has been available in an open-source package, SSMTTool [3], making direct SSM computations scalable to realistic, nonlinear finite-element models.

In this talk, we illustrate these computational advances on finite-element models of mechanical structures that range from academically simple models such as a beam to industrially complex models such as an aircraft wing.

## REFERENCES

- [1] Haller, G., & Ponsioen, S. (2016). Nonlinear normal modes and spectral submanifolds: existence, uniqueness and use in model reduction. *Nonlinear Dynamics*, 86(3), 1493–1534. <https://doi.org/10.1007/s11071-016-2974-z>
- [2] Jain, S., & Haller, G. (2021). How to Compute Invariant Manifolds and their Reduced Dynamics in High-Dimensional Finite-Element Models. *Nonlinear Dynamics*. <https://doi.org/10.1007/s11071-021-06957-4>
- [3] Jain, S., Thurnher, T., Li, M., Haller, G. (2021). SSMTTool: Computation of invariant manifolds in high-dimensional mechanics problems. *Zenodo*. <https://doi.org/10.5281/zenodo.4614201>