

# Estimating Approximate Control Variate Weights: with Applications in Importance Sampling and Rare Event Simulation

Alex Gorodetsky<sup>1</sup> and Trung Pham<sup>1</sup>

<sup>1</sup> Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI, USA,  
goroda@umich.edu

<sup>2</sup> Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI, USA,  
trungp@umich.edu

**Keywords:** *Multifidelity modeling, approximate control variates, importance sampling, rare events*

The recent growth in multi-fidelity uncertainty quantification has given rise to a large set of variance reduction techniques that leverage information from model ensembles to provide variance reduction for estimates of the statistics of a high-fidelity model. In this presentations we provide two contributions: (1) we utilize an ensemble estimator to account for uncertainties in the optimal weights of approximate control variate (ACV) approaches and derive lower bounds on the number of samples required to guarantee variance reduction; and (2) we extend an existing multi-fidelity importance sampling (MFIS) scheme to leverage control variates. Our approach directly addresses a limitation of many multifidelity sampling strategies that require the usage of pilot samples to estimate covariances. As such we make significant progress towards both increasing the practicality of approximate control variates—for instance, by accounting for the effect of pilot samples—and using multi-fidelity approaches more effectively for estimating low-probability events. The numerical results indicate our hybrid MFIS-ACV estimator achieves up to 50% improvement in variance reduction over the existing state-of-the-art MFIS estimator, which had already shown outstanding convergence rate compared to the Monte Carlo method, on several problems of computational mechanics.

## REFERENCES

- [1] Pham, T. and Gorodetsky, A. A., Ensemble approximate control variate estimators: Applications to multi-fidelity importance sampling. ArXiv 2021