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Interface Preconditioners for Multiphysics Problems

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We are interested in reliable simulations of interface-driven multiphysics problems that can be coupled across dimensions. Typical examples include geo- and biophysics, such as coupling of free flow and porous media flow or coupling of physical laws across dimensions. The coupling is imposed by the Lagrange multiplier that enforces constraints on the interface and ensures well-posedness in fractional Sobolev spaces weighted by material parameters. However, the complexity of the interface coupling often deteriorates the performance of standard preconditioners for finding the numerical solutions effectively.

Therefore, we develop preconditioning techniques that target specifically such multiphysics problems by exploiting fractional operators at the interfaces to achieve optimal performance in practice. Specifically, the robust preconditioners for the interface problems are represented as a sum of fractional Laplacians that can include both negative and positive fractionalities. To handle fractional operators numerically, we implement methods based on multilevel algorithms and rational approximations. We show that the proposed preconditioners are parameter-independent and scalable with regards to the number of the degrees of freedom of the system. We demonstrate the efficiency of our methods on several numerical examples of mixed-dimensional problems on realistic geometries, such as a 3D-1D model of flow in vascularized brain tissue.