The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

STABILIZATION OF THE HIGH-ORDER DISCRETIZED WAVE EQUATION FOR DATA ASSIMILATION PROBLEMS

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Keywords: *PDE Control, Discrete observability*

The objective of this work is to propose and analyze numerical schemes to solve data assimilation problems by observers for wave-like hyperbolic systems [1]. The observers strategies using available measurements are becoming popular to improve the quality of numerical simulations of physical phenomena.

The design of observers for these systems is now classical at the continuous level. The efficiency of these observers relies on the proof of the exponentially stable character of the underlying system when we add the dissipative term linked to the observations [2]. This exponential stability depends on observability inequalities where the energy of the system is proven to be controlled by the observations.

To demonstrate these inequalities, several methods exist in the literature: the multiplier method (introduced by J-L Lions as early as the 1980s) [3], spectral methods, Carleman estimates and micro-local analysis. For 1D problems, the multiplier approach is very natural and provides near-optimal results.

Unfortunately, the multiplier method is incompatible with the discretization needs. Indeed, without modification of the classical finite element methods used, the exponential stability (uniformly in h, the discretization step) is not preserved at the discrete level because of the presence of spurious waves [4].

The whole point of our work is therefore to propose correctors in the discretization that enable us to prove exponential stability at the discrete level when using high-order finite element approximation. The main idea is to add a stabilizing term which damps the oscillating functions (such as spurious waves). This term is built from a discrete multiplier analysis. This gives us the exponential stability of the semi-discrete problem at any order without affecting the order of convergence.

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