

Flow coupled with advection, reaction and diffusion in evolving porous media: homogenisation and simulation

David Wiedemann^{1,*}, Malte A. Peter²

¹ University of Augsburg, Universitätsstr. 14, 86159 Augsburg,
david.wiedemann@math.uni-augsburg.de, <https://appa.math.uni-augsburg.de/>

² University of Augsburg, Universitätsstr. 14, 86159 Augsburg,
malte.peter@math.uni-augsburg.de, <https://appa.math.uni-augsburg.de/>

Keywords: *Evolving porous medium, Darcy flow, Coupled advection–reaction–diffusion problem, Homogenisation*

Many biological processes in porous media cause a change of the microstructure, e.g. biofilm attachment or growth. This evolution affects strongly the effective permeability and often depends on a transported concentration. We model such a process with an advection–reaction–diffusion equation coupled to Stokes flow on the microscopic scale.

In order to derive mathematically an effective model, we transform the problem on a periodic substitute domain where we can pass rigorously to the homogenisation limit (cf. [1]). After a back-transformation, we obtain an effective homogenised model. It includes an advection–reaction–diffusion equation and a Darcy law for evolving microstructure (cf. [3, 2]). Furthermore, the coupling of the concentration with the domain evolution is transferred to a reference cell in the homogenisation limit. Thus, the effective diffusivity and permeability become time- and space-dependent and can be computed using the corresponding cell problems.

We illustrate the behaviour of the concentration, the local porosity and the local permeability by numerical simulations.

REFERENCES

- [1] D. Wiedemann, The two-scale transformation method. *ArXiv:2106.13730*, 2021.
- [2] D. Wiedemann and M. A. Peter, Darcy’s law for evolving microstructure. *Proc. Appl. Math. Mech.*, Vol. **21**, 2021.
- [3] D. Wiedemann and M. A. Peter, Homogenisation of the Stokes equations for evolving microstructure. *ArXiv:2109.05997*, 2021.