

# FINITE ELEMENT APPROXIMATION OF WAVE PROBLEMS WITH CORRECTING TERMS BASED ON TRAINING ARTIFICIAL NEURAL NETWORKS WITH FINE SOLUTIONS

Arnaud Fabra<sup>§‡\*</sup>, Joan Baiges<sup>§</sup> and Ramon Codina<sup>§‡</sup>

<sup>§</sup> Universitat Politècnica de Catalunya, Barcelona Tech, Jordi Girona 1-3, Edifici C1, 08034 Barcelona, Spain.

<sup>‡</sup> Centre Internacional de Mètodes Numèrics en Enginyeria (CIMNE), Edifici C1, Campus Nord UPC, Gran Capità S/N, 08034 Barcelona, Spain.

E-mails: afabra@cimne.upc.edu (AF), joan.baiges@upc.edu (JB), ramon.codina@upc.edu (RC)

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In this work we present a general idea, based on Reduced Order Models, to correct coarse models by introducing a correcting term designed from fine solutions [1]. In order to create this correcting term, we solve a numerical case with two different discretizations, a coarse and a fine one. Then, we make use of a training algorithm to compare the solutions of the fine and coarse cases. With the obtained information, the training algorithm is able to create a correction term that is then added to the coarse approximation in following simulations, thus yielding results closer to the ones that would be obtained with the fine discretization.

Regarding the correction model, we explore two options. On the one hand, a classic Least Squares model is used; it turns out to work well in simple models but it has limitations in complex scenarios. On the other hand, Artificial Neural Networks are used; their flexibility and capacity to adapt to almost every model being their main feature.

We apply this idea to solve the wave equation in the time and frequency domains. To do that, we employ a Finite Element formulation that belongs to the computational framework of Variational Multi-Scale methods [2]. The performance of this method is tested through four different acoustic numerical examples, where the fine solutions are characterized for having a finer discretization either in time or space.

## REFERENCES

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