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A GENERALIZED MULTIGRID METHOD FOR CONTACT PROBLEMS IN LAGRANGE MULTIPLIER BASED UNFITTED FINITE ELEMENT METHOD

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Internal interfaces in a domain could exist as a material defect or they can appear due to propagations of cracks. Discretization of such geometries and solution of contact problems on the internal interfaces can be computationally challenging. We employ an unfitted Finite Element (FE) framework for the discretization of the domains and develop a tailored, globally convergent, and efficient multigrid method for solving contact problems on the internal interfaces. In the unfitted FE methods, structured background meshes are used and only the underlying finite element spaces are modified to incorporate the discontinuities. The non-penetration conditions on the embedded interfaces of the domains are discretized using the method of Lagrange multipliers. We propose a tailored multigrid algorithm that can tackle the arising variational inequality problem. Our multigrid method employs L^2-projection-based transfer operators to construct a hierarchy of nested FE spaces in the unfitted FE framework. In addition, we introduce a technique to decouple discrete non-penetration constraints by means of an orthogonal transformation. These decoupled constraints are handled by a variant of a projected Gauss-Seidel method, which we employ as a smoother in the multigrid method. These components of the multigrid method allow us to enforce linear constraints locally and ensure global convergence. We will demonstrate the robustness, efficiency, and level independent convergence property of the proposed method for Signorini's problem and two-body contact problems.

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