

## Efficient time marching methodologies for a p-adaptive Discontinuous Galerkin approximation

Gerasimos Ntoukas<sup>1a</sup>, Wojciech Laskowski<sup>1</sup>, Gonzalo Rubio<sup>1,2</sup> and Esteban Ferrer<sup>1,2</sup>

<sup>1</sup> ETSIAE-UPM - School of Aeronautics, Universidad Politécnica de Madrid,  
Plaza Cardenal Cisneros 3, E-28040 Madrid, Spain,  
email<sup>a</sup>: gerasimos.ntoukas@upm.es

<sup>2</sup> Center for Computational Simulation, Universidad Politécnica de Madrid,  
Campus de Montegancedo, Boadilla del Monte, 28660 Madrid, Spain

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The objective of this work is to present modern acceleration techniques for different types of flows in the context of higher-order-methods. Multigrid methods constitute a significant component for the speed-up of numerical simulations. Fast steady-state solvers are essential for various types of fluid flow solvers (e.g., RANS solvers or unsteady flows based on dual time stepping that can be used for compressible Navier-Stokes or multiphase flow applications). Some of the most popular and efficient techniques up to date consist of coarse grid acceleration in combination with semi-implicit smoothers [1]. More specifically, for higher-order-methods, a natural choice is p-multigrid since the coarser levels are obtained by reducing the approximation order of the solution [2]. The numerical framework in this work consists of a compressible Navier-Stokes solver complimented with the Spalart-Allmaras turbulence model, approximated with a high-order discontinuous Galerkin spectral element method. We analyze the performance of several popular relaxation strategies for multigrid: explicit Runge-Kutta methods with local time stepping, smoothers with local or block LU factorization (e.g. block-Jacobi) and smoothers with incomplete factorization (e.g. ILU(k)) on representative test cases, such as the flow past an airfoil. In addition, through the method of p-adaptation, a characteristic of higher-order-methods, we aim to further accelerate the simulation and reduce the associated computational cost through element-local adaptation of the polynomial order [3].

### References

- [1] Antonio Ghidoni, Alessandro Colombo, Francesco Bassi, and Stefano Rebay. Efficient p-multigrid discontinuous galerkin solver for complex viscous flows on stretched grids. *International Journal for Numerical Methods in Fluids*, 75(2):134–154, 2014.
- [2] Krzysztof J Fidkowski, Todd A Oliver, James Lu, and David L Darmofal. p-multigrid solution of high-order discontinuous galerkin discretizations of the compressible navier-stokes equations. *Journal of Computational Physics*, 207(1):92–113, 2005.
- [3] Andrés M Rueda-Ramírez, Juan Manzanero, Esteban Ferrer, Gonzalo Rubio, and Eusebio Valero. A p-multigrid strategy with anisotropic p-adaptation based on truncation errors for high-order discontinuous galerkin methods. *Journal of Computational Physics*, 378: 209–233, 2019.