

## A block preconditioner for equilibrium equations in mechanics with a second gradient of dilatation regularization

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We are interested in the modelling of a deep geological disposal facility built in a clay-based host rock. To avoid problems with the loss of uniqueness of a solution and, more importantly, problems of localizations which are often encountered in soil computations, we consider non-locally regularized equations based on a second gradient theory [1]. In this method, a new primal unknown, modelling the trace of the displacement gradient, is introduced. The objective of this work is the development of a scalable solution technique for the linearized equation.

There has been research on models involving second gradient regularizations, but linear solvers have not been in the focus of these works [2]. In this presentation, we will present a block preconditioner for the mechanics equilibrium equations with a regularization via a second gradient of dilation. Our proposed preconditioner is based on the theory of block preconditioners for saddle point problems [3]. We use a block Jacobi approach, where we precondition with an algebraic multigrid the blocks corresponding to the finite element discretization of the displacement and micro volume changes, whereas the Lagrange multipliers block is preconditioned with a weighted mass matrix.

The method is implemented in Firedrake [4]. We present numerical results that reflect the good performance of the proposed preconditioner. In particular, when used inside a fgmres method, the number of iterations until convergence is independent of the mesh size.

### REFERENCES

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