

# Investigation of the Navier–Stokes–Cahn–Hilliard diffuse interface model for numerical simulations of unstable liquid filaments

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**Keywords:** *Navier–Stokes–Cahn–Hilliard, diffuse-interface, finite element method, adaptivity*

Thin, long, columns of liquid, called fluid filaments, occur in many industrial applications. Examples are, the extrusion process in additive manufacturing and the ink ejection in inkjet printing. Fluid filaments show complex dynamics such as contraction, oscillations, and potentially also topological changes. The dynamics of these filaments play an important role in the droplet formation within the inkjet printing process. Stable filaments contract to a single droplet, while unstable filaments break up into droplets and new, smaller, filaments (Anthony, Kamat, Harris, & Basaran, 2019). This cascading process affects the droplet size distribution, which influences the print quality. Minuscule droplets, called satellite droplets, can be detrimental to the reliability of the printing process and to print quality (Wijshoff, 2010).

In this work, we use the Navier–Stokes–Cahn–Hilliard (NSCH) diffuse-interface model (Abels, Garcke, & Grün, 2012) to predict the dynamics of fluid filaments. The NSCH phase-field model provides an inherent description of topological changes in the filament. To enable the consideration of diffuse interfaces that are significantly smaller than other characteristic length scales in the problem under consideration, we approximate the NSCH equations by means of a finite-element method with adaptive spatial resolution based on an a-posteriori error estimate. In addition, we apply an adaptive temporal discretization. We use an  $\epsilon$ -continuation for robust spatial refinements (Van Brummelen, Demont, & van Zwieten, 2020). We investigate the influence of the two main model parameters in the NSCH equations, viz. the interface-thickness parameter  $\epsilon$  and the mobility parameter  $m$ , on a filament test case. This test case comprises an infinitely long filament which succumbs to a Rayleigh–Plateau instability. The results are compared to two sharp interface reference simulations. Specifically, we evaluate the shape of breakup and breakup time of the filament.

## References

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