

Mass Conserving Mixed Stress-Strain rate Finite Element Methods for Non-Newtonian Fluid Simulations

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Keywords: *Non-Newtonian flows, Incompressible flows, Mixed finite element methods*

Many non-Newtonian models assume a non-linear relation between the deviatoric stress tensor τ and the rate-of-strain tensor $\varepsilon(u)$, which is not necessarily given in explicit form. Therefore the requirement on a finite element method is the capability to capture the behavior of the non-linear constitutive relation.

Inspired by the work of [1] and assuming incompressible, stationary, isothermal, laminar flow, we present a new mixed finite element method based on the *Hu-Washizu principle* known from elasticity. Introducing a variable for the rate-of-strain tensor $\varepsilon(u)$ allows a natural embedding of a general implicit constitutive relation of the form $G(\tau, \varepsilon(u)) = 0$, and $H(\text{div})$ -conforming velocity approximations results in exact conservation of mass.

We discuss solvability of the novel formulation and present several numerical examples with various constitutive laws.

REFERENCES

- [1] J. Gopalakrishnan, P. L. Lederer, and J. Schöberl, A mass conserving mixed stress formulation for the Stokes equations, *IMA Journal of Numerical Analysis*, Vol. **40**, Iss. 3, pp. 1838-1874, 2020.