

## Energy-conserving discretization of the pressure-free two-fluid model for one-dimensional flow in ducts

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**Keywords:** *multiphase flow, energy-conserving discretization, finite volume method*

The pressure-free two-fluid model (PFTFM) [1] is a recent reformulation of the one-dimensional two-fluid model (TFM) for stratified flow in ducts, in which the pressure is eliminated through intricate use of the volume constraint. This removes the pressure Poisson equation and all constraints from the model formulation, making the PFTFM more amenable to analysis, at the cost of requiring knowledge of the volumetric flow rate. For the discrete initial boundary value problem, this eliminates the need for solving the pressure Poisson equation, and significantly reduces the computational cost.

In this work, we show the novel result that this pressure-free model can be formulated in an energy-consistent manner, and we determine how to choose the free parameter of the model (the volumetric flow rate) in such a way that the model is momentum and energy-conserving.

Furthermore, we extend our energy-conserving spatial discretization of the TFM [2], in the form of a finite volume scheme, to the PFTFM. We propose a specific discretization of the volumetric flow rate that yields discrete momentum and energy conservation.

Our numerical experiments confirm that the discrete energy is conserved for different problem settings, including sloshing in an inclined closed tank, and a traveling wave in a periodic domain. The solution matches the TFM solution closely, with reduced computation time, and with more precise momentum and energy conservation. Furthermore, having removed the non-conservative pressure term while keeping energy conservation, the new model is an important step towards an entropy-stable formulation of the TFM.

## REFERENCES

- [1] Sanderse, B. et al. A novel pressure-free two-fluid model for one-dimensional incompressible multiphase flow. *Journal of Computational Physics* (2021) **426**:109919.
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