The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

Analysis of topological derivative for qualitative identification using elastic waves

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Keywords: *Topological derivative, Inverse problem, Inverse scattering, Elastic waves*

The concept of topological derivative has proved effective as a qualitative inversion tool for wave-based identification of finite-sized objects, see e.g. [5]. Although this approach is usually based on a heuristic interpretation of the topological derivative, several contributions towards its mathematical justification are available [1, 2, 3, 6, 7]. This work extends previous efforts in [2, 3], dealing with scalar wave problems, to the identification of elastic inhomogeneities embedded in elastic media interrogated by elastic waves. The data used for identification, assumed to be of near-field nature (i.e. no far-field approximation is introduced), is introduced through a misfit functional J. The imaging functional that reveals embedded inhomogeneities then consists of the topological derivative $\mathcal{T}_J$ of J (in particular, the actual minimization of J is not performed, and therein lies the main source of computational savings relative to standard inversion based on PDE-constrained minimization). Here, the main contribution consists in an analysis of $\mathcal{T}_J$ using a suitable factorization of the near fields, achievable thanks to a recently-available convenient reformulation of the volume integral equation formulation of the forward elastodynamic scattering problem [4]. Our results include justification of both the sign heuristics for $\boldsymbol{z} \mapsto \mathcal{T}_J(\boldsymbol{z})$ (which is expected to be most negative at points $\boldsymbol{z}$ inside, or close to, the support of the sought flaw) and the spatial decay of $\mathcal{T}_J(\boldsymbol{z})$ as $\boldsymbol{z}$ moves away from the flaw support. This result, although being subject to a limitation on the strength of the inhomogeneity to be identified, provides a theoretical validation of the usual heuristic interpretation of $\mathcal{T}_J$ as an imaging functional. Our theoretical findings will be validated and demonstrated on 3D elastodynamic computational experiments.

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