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THIRD-ORDER TIME INTEGRATION SCHEME FOR DYNAMIC ANALYSIS OF COSSERAT RODS

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In many numerical formulations of three-dimensional beams the configuration space consists of three-dimensional rotations. In Cosserat beam model rotational degrees of freedom describe the orientations of cross-sections, which are allowed to rotate about the centroid attached to the reference axis. In our formulation, we employ the algebra of quaternions as computationally most suitable representation of rotations. Such representation is free from singularities, while only a single scalar constraint needs to be satisfied for any rotational quaternion.

We will present a time integration scheme of third-order that is properly adapted to quaternion representation of rotations and the governing equations of Cosserat rods. Our scheme exactly preserves the unit norm constraint of rotational quaternions following the approach of Munthe-Kaas [1] and Zanna [2] and adopting it to quaternion algebra. The correction function needed to compensate the non-commutativity is derived to introduce an implicit time integration scheme of the third order. The proposed method consists of two implicit stages of second order followed by an explicit third-order step, which allows local error control without any additional computational costs. The implicit scheme leads to a system of nonlinear algebraic equations, where the rotations at the next time step need to be evaluated iteratively. An efficient approach for linearization and update procedure will also be presented. The computational behaviour of proposed method will be demonstrated by several examples.

REFERENCES

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