

Convergence Analysis for some AVEMs

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The task of building a mathematically sound theory for Adaptive Virtual Element Methods (AVEMs) is a challenging task. Among the realm of polygonal meshes, we restrict our analysis to triangular meshes with hanging nodes in 2d – the simplest meshes with a systematic refinement procedure that preserves shape regularity and optimal complexity.

A major challenge in the a posteriori error analysis of AVEMs is the presence of the stabilization term, which is of the same order as the residual-type error estimator but prevents the equivalence of the latter with the energy error. Under the assumption that any chain of recursively created hanging nodes has uniformly bounded length, we show that the stabilization term can be made arbitrarily small relative to the error estimator provided the stabilization parameter of the scheme is sufficiently large. This quantitative estimate leads to stabilization-free upper and lower a posteriori bounds for the energy error.

Based on these results, a 2-loop adaptive algorithm is defined, and its convergence properties are established via a contraction argument. The optimality properties of the algorithm are also discussed. Our results apply to H^1 -conforming VEMs of any kind, including the classical and enhanced VEMs.

REFERENCES

[1] Beirão da Veiga L., Canuto C., Nochetto R.H., Vacca G., Verani, M., *A-VEM: Stabilization-Free A Posteriori Error Analysis*, arXiv:2111.07656 [math.NA] 15 Nov 2021.