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ON SPACE-TIME FINITE ELEMENT APPROXIMATIONS OF THE DYNAMIC BIOT SYSTEM

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The numerical simulation of the (prototype) dynamic Biot system of poroelasticity

$$\rho\partial_t^2 \boldsymbol{u} - \boldsymbol{\nabla}\cdot(\boldsymbol{C}\boldsymbol{\varepsilon}(\boldsymbol{u})) + \alpha\boldsymbol{\nabla} p = \rho\boldsymbol{f}\,, \tag{1a}$$

$$c_0\partial_t p + \alpha\boldsymbol{\nabla}\cdot\partial_t\boldsymbol{u} - \boldsymbol{\nabla}\cdot(\boldsymbol{K}\boldsymbol{\nabla} p) = g\,, \tag{1b}$$

supplemented with appropriate initial and boundary conditions, has become of importance in several branches of natural sciences and technology for analyzing experimental data or designing theories and therapies based on mathematical concepts. Thermoelasticty also offers numerous applications of the model (1). The mixed hyperbolic-parabolic character of (1) evokes substantial challenges for its reliable and efficient discretization.

A natural and promising approach for the numerical approximation of the coupled system (1) is given by the application of space-time finite element methods (cf. [1, 2, 3]) that are based on variational formulations in time and space. Here we study the approximation of (1) by families of continuous, enriched and discontinuous finite element methods in space and continuous and discontinuous approximations of the time variable within a monolithic approach. Error estimates and numerical experiments are presented.

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