

## Space-Time Finite Element Methods for Flow Problems

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Space-time finite element methods provide a combined discretization of space and time for the numerical solution of partial differential equations (PDEs). In this contribution, we highlight advantages of space-time discretizations for simulations of time-dependent flow problems [1].

As a prototype test, the numerical convergence behavior of time-discontinuous space-time finite element methods is investigated for a scalar advection-diffusion model problem. Therein, the model parameter choices include the parabolic and hyperbolic limit cases. The computational error analysis demonstrates temporal superconvergence of prismatic space-time finite elements for parabolic problems.

After tests of the prototype, this contribution focuses on high-performance computing (HPC) applications of simplex space-time meshes. For example, a spatial computational domain with time-varying topology can be discretized with a connected boundary-conforming space-time mesh. Application cases include the flow through a closing and opening valve and through the geometry of a clamped artery. In these cases, a three-dimensional spatial geometry requires a four-dimensional pentatope mesh with a very large number of elements [2]. The advanced meshing capabilities also open up a path to parallel-in-time computations on complex domains. In the presented simulations, domain-decomposition is not only applied to the spatial domain, but to the complete space-time domain.

## REFERENCES

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