

Parameter-robust monolithic solvers for Stokes-Darcy/Biot systems

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Keywords: *Multiphysics problems, Operator preconditioning, Perturbed saddle-point problems*

Systems exhibiting free flow coupled with flow in porous (and deformable) media, as described by coupled Stokes-Darcy/Biot models, arise in numerous environmental, industrial and medical applications. For formulations where the coupling between the components is enforced by the Lagrange multipliers, efficient and parameter-robust solvers of the resulting multiphysics systems, rely on operators in fractional Sobolev spaces defined over the interface [2]. This property is arguably to be expected due to the presence of the explicit coupling variable.

In this talk we discuss formulations of the Stokes-Darcy [1, 3] and Stokes-Biot [4] models which are free of Lagrange multipliers. We establish well-posedness of the systems in a unifying theoretical framework and in turn derive parameter-robust solvers. Despite the absence of the explicit interface variable we show that fractional order operators are again a crucial component for achieving the robustness property.

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