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AUGMENTED LAGRANGIAN BLOCK PRECONDITIONERS FOR INCOMPRESSIBLE RESISTIVE MAGNETOHYDRODYNAMICS

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We consider the incompressible viscoresistive magnetohydrodynamics (MHD) equations:

$$\begin{aligned}
-\frac{2}{\mathrm{Re}}\operatorname{div}\varepsilon(\mathbf{u}) + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla p + S\,\mathbf{B}\times(\mathbf{E}+\mathbf{u}\times\mathbf{B}) &= \mathbf{f},\\
\operatorname{div}\mathbf{u} &= 0,\\
\mathbf{E}+\mathbf{u}\times\mathbf{B} - \frac{1}{\mathrm{Re_m}}\,\mathbf{curl}\,\mathbf{B} &= \mathbf{0},\\
\mathbf{curl}\,\mathbf{E} &= \mathbf{0},\\
\operatorname{div}\mathbf{B} &= 0.
\end{aligned}$$

Here, $\mathbf{u} : \Omega \to \mathbb{R}^3$ denotes the velocity, $p : \Omega \to \mathbb{R}$ the fluid pressure, $\mathbf{B} : \Omega \to \mathbb{R}^3$ the magnetic field, $\mathbf{E} : \Omega \to \mathbb{R}^3$ the electric field, Re the fluid Reynolds number, $\mathrm{Re_m}$ the magnetic Reynolds number, S the coupling number, $\mathbf{f} : \Omega \to \mathbb{R}^3$ a source term and $\varepsilon(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^\top)$.

These equations are generally known to be difficult to solve numerically. They are highly nonlinear and exhibit strong coupling between the electromagnetic and hydrodynamic variables, especially for high Reynolds and coupling numbers.

In this work, we present a scalable augmented Lagrangian preconditioner for a finite element discretization of these equations. For stationary problems, our solver achieves robust performance with respect to the Reynolds and coupling numbers in two dimensions and good results in three dimensions. We extend our method to fully implicit methods for time-dependent problems which we solve robustly in both two and three dimensions. Our approach relies on specialized parameter-robust multigrid methods for the hydrodynamic and electromagnetic blocks. The scheme ensures exactly divergence-free approximations of both the velocity and the magnetic field up to solver tolerances.

We confirm the robustness of our solver by numerical experiments in which we consider fluid and magnetic Reynolds numbers and coupling numbers up to 10,000 for stationary problems and up to 100,000 for transient problems in two and three dimensions.