The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5 – 9 June 2022, Oslo, Norway

Snap-through instability during transmission of rotation by a flexible shaft with intrinsic curvature

Yury Vetyukov[1]*, Evgenii Oborin[1]

[1] TU Wien, Getreidemarkt 9, 1060 Vienna, Austria
yury.vetyukov@tuwien.ac.at, https://www.mec.tuwien.ac.at/staff/mechanics_of_solids/EN/

Key Words: *Spatial theory of rods, Geometric nonlinearity, Incremental equations, Nonlinear finite element analysis, Bifurcation of equilibria*

Various technical solutions feature flexible shafts to transmit rotary motion from a remote motor, the most prominent example being a drillstring rotating in a borehole. The intrinsic curvature of the shaft, whose undeformed configuration is thus not straight, and the curved spatial line of the borehole result into a non-uniform rotation of the driven end even if the rotation angle at the driving end grows slowly at a constant time rate [1]. The reason for such behaviour is clear: the configurations, in which the own curvatures of the shaft and of the borehole match better, are energetically more advantageous, and one needs to twist the shaft more to rotate it through an energetically less advantageous state. The available analytical models assume the shaft to be a flexible rod, which is firmly fitted in a rigid tube without friction. The dependence of the rotation angle of the driven end on the one of the driving end follows from the numerical integration of a boundary value problem [1] or analytically in terms of elliptic functions for special shapes of the rod and the tube [2]. This allows finding the critical regions of parameters, in which the dependence between the two rotation angles becomes non-unique, thus meaning the onset of a snap-through instability: the rotation at the driven end becomes essentially non-uniform with jumps. Depending on the specific technical application, the effect may be either regarded as dangerous or be the desired one. Similar behaviour shall be observed for a straight shaft with an unsymmetric cross-section, such that its bending stiffness coefficients become unequal.

In the present contribution we consider the rod being freely deformable in space, thus releasing the constraint regarding its shape. The positions and tangential directions in the end points are, however, prescribed by the boundary conditions, such that the bent shaft is fitted into revolute joints at its ends. This additional freedom of changing the spatial shape of the rod in the course of transmitting the rotation makes analytical solutions possible only for the linearized equations, when the intrinsic curvature of the rod is considered infinitesimally small and the incremental form of the geometrically nonlinear theory of rods is applied. The reference solutions are obtained with a spatial rod finite element model featuring bending and torsion [3].

REFERENCES

[1] A.K. Belyaev and V.V. Eliseev, Flexible rod model for the rotation of a drill string in an arbitrary borehole. *Acta Mechanica*, Vol. **229**(2), pp. 841–848, 2018.

[2] V.V. Eliseev and T.V. Zinovieva, On transmission of rotation by a flexible shaft. *J. Theory Mach. Mech*, Vol. **3**(2), pp. 67–72, 2005 (in Russian).

[3] P.G. Gruber, K. Nachbagauer, Y. Vetyukov and J. Gerstmayr, A novel director-based Bernoulli–Euler beam finite element in absolute nodal coordinate formulation free of geometric singularities. *Mechanical Sciences* Vol. **4**(2), 279–289, 2013.