

## Matrix-free, hybridised, compatible, high order finite element methods in Firedrake

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One way to achieve high accuracy for simulations of physically complex problems is to discretise the partial differential equations with higher order, compatible finite element methods (FEM). Since higher order, compatible approximations result in big equation systems, performance optimisations become crucial. A speedup of the system solve can be established by loosening the global coupling of the FEM with a hybridization preconditioner, such that it is sufficient to execute expensive operations on smaller matrices. The preconditioner is defined through local linear algebra operations on FEM tensors and is represented in the domain specific language Slate in the Finite Element software framework Firedrake.

The local linear algebra operations need further optimisations in order to achieve high performance for high order, compatible FEM, because not only the size of the global but also the local tensors is considerably high. By employing locally matrix-free methods high storage requirements/data movement can be avoided in favor of executing more FLOPS and a high FLOP/data ratio is advantageous for high performance on recent computer architectures.

I will talk about the automatic code generation of fully matrix-free, hybridised, compatible, high order FEM in Firedrake.