

## Dynamic phase-field fracture with a first-order discontinuous Galerkin method for elastic waves

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**Keywords:** *Phase-field fracture, elastic waves, first-order DG method*

We present a new numerical method for dynamic fracture at small strains which is based on a discontinuous Galerkin approximation of a first-order formulation for elastic waves [2] and where the fracture is approximated by a phase field driven by a stress based fracture criterion [4]. The staggered algorithm in time combines the implicit midpoint rule for the wave propagation followed by an implicit Euler step for the phase field evolution. Then, driven by the fracture criterion, the material is degraded, and the waves are reflected at the diffusive interface. We demonstrate in 2D and 3D applications the dynamic fracture evolution with multiple fractures initiated by reflections.

Finally we discuss the extension to visco-elastic materials and different energy based elastic driving forces as it is analyzed in [3]. Here we show that the first-order approach extends to visco-elastic materials [1] and remains stable also in the limit of very small viscosity.

## REFERENCES

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